23. 解：$\int_0^1 \sqrt[3]{1+x}\,dx = \int_0^1 (1+x)^{\frac{1}{3}} d(1+x)$

$$= \frac{3}{4}(1+x)^{\frac{4}{3}}\Big|_0^1$$

$$= \frac{3}{4}(2\sqrt[3]{2} - 1).$$

24. 解：$\int x \sin x\,dx = -x\cos x + \int \cos x\,dx$

$$= -x\cos x + \sin x + C.$$

25. 解：所求体积为

$$V = \pi \times 4^2 \times 2 - \int_0^2 \pi x^2\,dy$$

$$= 32\pi - \pi\int_0^2 y^4\,dy = \frac{128\pi}{5}.$$

26. 解：由已知得

$y' = 3x^2 - 6x + 2$,

$y'' = 6x - 6$.

令 $y'' = 0$，得 $x = 1$. 当 $x < 1$ 时，$y'' < 0$，当 $x > 1$ 时，$y'' > 0$，故曲线 $y = x^3 - 3x^2 + 2x + 1$ 的凸区间为 $(-\infty, 1)$，凹区间为 $(1, +\infty)$，拐点为 $(1,1)$.

27. 解：(1) 由题设知 $a + 0.5 + b = 1$.

由 $EX = 0$，得 $(-1) \times a + 0 \times 0.5 + 2 \times b = 0$，即 $a = 2b$.

解得 $a = \frac{1}{3}, b = \frac{1}{6}$.

$$(2)E(X(X+1)) = (-1) \times (-1+1) \times \frac{1}{3} + 0 \times (0+1) \times \frac{1}{2} + 2 \times (2+1) \times \frac{1}{6}$$

$$= 1.$$

28. 解：由已知得

$$\frac{\partial z}{\partial x} = 2x + 4y^2 - 2, \frac{\partial z}{\partial y} = 8y^3 + 8xy,$$

$$\frac{\partial^2 z}{\partial x^2} = 2, \frac{\partial^2 z}{\partial x \partial y} = 8y, \frac{\partial^2 z}{\partial y^2} = 24y^2 + 8x.$$

令 $\frac{\partial z}{\partial x} = 0, \frac{\partial z}{\partial y} = 0$ 得 $\begin{cases} x = -1, \\ y = 1, \end{cases} \begin{cases} x = -1, \\ y = -1, \end{cases} \begin{cases} x = 1, \\ y = 0. \end{cases}$

记 $A = \frac{\partial^2 z}{\partial x^2}, B = \frac{\partial^2 z}{\partial x \partial y}, C = \frac{\partial^2 z}{\partial y^2}$. 在点 $(-1,1)$ 及点 $(-1,-1)$ 处，$AC - B^2 < 0$，故点 $(-1,1), (-1,-1)$ 不是极值点. 在点 $(1,0)$ 处，$A > 0, AC - B^2 > 0$，故点 $(1,0)$ 为极小值点，极小值为 -1.