

三、解答题

$$\begin{aligned}
 21. \text{解: } \lim_{x \rightarrow 0} \frac{1 - \cos x - x^2}{2 \sin^2 x} &= \lim_{x \rightarrow 0} \frac{\sin x - 2x}{4 \sin x \cos x} \\
 &= \lim_{x \rightarrow 0} \frac{\sin x}{4 \sin x \cos x} - \lim_{x \rightarrow 0} \frac{2x}{4 \sin x \cos x} \\
 &= -\frac{1}{4}.
 \end{aligned}$$

$$\begin{aligned}
 22. \text{解: } f'(x) &= e^x (\cos x - \sin x), \\
 f''(x) &= -2e^x \sin x, \\
 f''\left(\frac{\pi}{2}\right) &= -2e^{\frac{\pi}{2}}.
 \end{aligned}$$

$$\begin{aligned}
 23. \text{解: } \int_0^1 \sqrt[3]{1+x} dx &= \int_0^1 (1+x)^{\frac{1}{3}} d(1+x) \\
 &= \frac{3}{4} (1+x)^{\frac{4}{3}} \Big|_0^1 \\
 &= \frac{3}{4} (2\sqrt[3]{2} - 1).
 \end{aligned}$$

$$\begin{aligned}
 24. \text{解: } \int x \sin x dx &= -x \cos x + \int \cos x dx \\
 &= -x \cos x + \sin x + C.
 \end{aligned}$$

25. 解: 特征方程为 $r^2 - r - 2 = 0$, 解得特征根 $r_1 = -1, r_2 = 2$.
所以微分方程的通解为 $y = C_1 e^{-x} + C_2 e^{2x}$.

26. 解: 由已知得

$$\begin{aligned}
 y' &= 3x^2 - 6x + 2, \\
 y'' &= 6x - 6.
 \end{aligned}$$

令 $y'' = 0$, 得 $x = 1$. 当 $x < 1$ 时, $y'' < 0$, 当 $x > 1$ 时, $y'' > 0$, 故曲线 $y = x^3 - 3x^2 + 2x + 1$ 的凸区间为 $(-\infty, 1)$, 凹区间为 $(1, +\infty)$, 拐点为 $(1, 1)$.

$$\begin{aligned}
 27. \text{解: } f(x) &= \frac{1}{2+x} \\
 &= \frac{1}{3} \cdot \frac{1}{1 + \frac{1}{3}(x-1)} \\
 &= \sum_{n=0}^{\infty} \frac{(-1)^n}{3^{n+1}} (x-1)^n.
 \end{aligned}$$

其收敛区间为 $(-2, 4)$.

$$\begin{aligned}
 28. \text{解: } \iint_D xy dx dy &= \int_0^{\frac{\pi}{4}} d\theta \int_0^1 r^2 \cos \theta \sin \theta \cdot r dr \\
 &= \frac{1}{2} \sin^2 \theta \Big|_0^{\frac{\pi}{4}} \cdot \frac{1}{4} r^4 \Big|_0^1 = \frac{1}{16}.
 \end{aligned}$$